Frequency Response Measurements with the E1437A

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There are many techniques for measuring the frequency response of a linear system. Some determine only the magnitude of the frequency response while others determine both the magnitude and phase of the response. In either case the measurement requires a signal to be applied to the system under test and a measurement of the response. Some techniques apply a sinusoidal signal which is swept or stepped across the frequency range of interest. Others apply a repetitive signal containing many discrete frequencies at which the response can be measured simultaneously without sweeping. Still others apply a transient which contains energy at all frequencies within the band of interest. The swept or stepped techniques can achieve very accurate results but are typically relatively slow. The multi-tone technique can be much faster but requires a specialized source to generate the appropriate repetitive signal. The transient excitation method can be very fast, is conceptually simple, and requires no special sources, but it is seldom used for precision measurements because of the poor signal-to-noise ratio typically achievable with transient excitation.

A significant benefit of the multi-tone and transient techniques is that the relative phase versus frequency can be obtained without a separate reference channel. That is because the excitation signal applies all signal frequencies simultaneously with a known initial phase relationship. Thus, the receiver can determine relative phase shifts at different frequencies without a separate phase reference. To obtain phase information with the swept or stepped approach requires a phase reference channel. This reference can sometimes be hidden if the source and receiver are contained in the same instrument or provide some sort of external phase locking mechanism. End-to-end frequency response measurements, where a reference channel is not available, typically use a multi-tone or transient excitation.

Most electrical engineers encounter the concept of frequency response in a sophomore level course on linear circuits. The topic is usually introduced with the concepts of superposition, impulse response, convolution, and step response. Although impulse and step responses remain as key mathematical representations for linear circuits, practical measurement limitations usually dictate that linear circuits be measured and characterized by their response to sinusoidal signals, with the impulse and step response being computed from this frequency response. Clearly, generating an ideal impulse source is not practical and most devices under test do not retain their linearity when exposed to an infinite voltage applied for zero time. However, the classical step function can be approximated with available sources and for many applications can be used as a practical excitation.

One disadvantage of using a step excitation is the fact that it's spectral energy decreases at high frequency with a slope of 20dB/decade. The size of the step must be kept small enough to be within the linear operating range of the system under test and of the measurement receiver connected to the output. The result is a potentially low signal to noise ratio at high frequency, and the computed frequency response will have poor accuracy at high frequency. A second disadvantage of step excitation is that slight nonlinearities in the response measurement can alter the apparent step response, resulting in an erroneous frequency response.

With its low noise and excellent linearity, the HP E1437A can overcome both of these problems and provide good results for direct step response characterization of linear systems. Since a complete frequency response graph can be generated from an individual voltage step, the update rate is very rapid. This is particularly useful for applications requiring real time feedback of frequency response, such as flatness adjustment, or channel adaptation. In many cases the normal signal encountered by the system consists of a sequence of voltage steps. Thus, the step excitation is actually a better signal than a sine wave for characterizing the system, since it is more similar to the typical signal and does not require the system to operate outside its normal conditions.

Using the E1437A to measure frequency response with a step excitation involves the following steps.

1. Apply a repetitive step signal (a square wave) with sufficient period to avoid overlapping the system step responses.

2. Set up the E1437A with the bandwidth (and sample rate) high enough to cover the frequency range over which the frequency response is to be

measured. Make sure the analog alias filter is turned on.

3. Trigger and collect a block of data which encompasses the complete step response.

4. Approximate the derivative of the response as the difference between adjacent samples. $y(n) = x(n+1) - x(n)$

5. Retain only the portion of this result which contains the system impulse response. Set to zero all samples outside this region to reduce random noise in the result.

6. Append or remove zeroed samples to achieve a block length which is an integer power of two. The sample density of the frequency response can be controlled by selecting the appropriate block length here.

7. Compute an FFT of the data to achieve the preliminary frequency response. Do not apply a weighting function or "window" the data before computing this transform.

8. If measurement noise causes too much variance in the frequency response, repeat steps 3-7 and average the response functions to achieve the desired variance.

9. Compensate for the effect of using the finite difference approximation of the derivative by multiplying the frequency response function by $\pi f / f_s \sin(\pi f / f_s)$, where f_s is the sample frequency.

10. If maximum accuracy is required, compensate for the known frequency response of the E1437A digital and analog filters.

Figure-1 shows actual measured data collected with an E1437A. The voltage step was from a 3V peak-to-peak, 1kHz square wave generated by an HP 8131A Pulse Generator. The E1437A bandwidth was set to 1MHz, and the sample rate was set to 2.56MSample/sec.

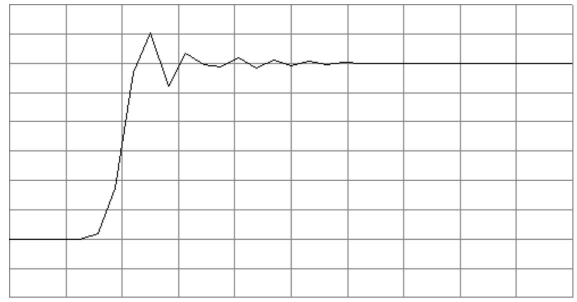

Figure-1, Step response 0.5V/div vertical, 1.25us/div horizontal

The graph shows adjacent samples connected by straight lines. This is not an accurate representation of the underlying continuous signal between samples. This does not detract from the fact that the sample values themselves are valid and contain sufficient information to reconstruct the continuous step response if we wanted to. However, we do not need to reconstruct the continuous step response in order to compute the frequency response of the system.

Figure-2 shows the time record resulting from computing the difference of adjacent samples from the step response. Again, the straight lines drawn between points do not accurately represent the underlying continuous step response. Again, we don't care since the complete signal information is contained in the discrete samples.

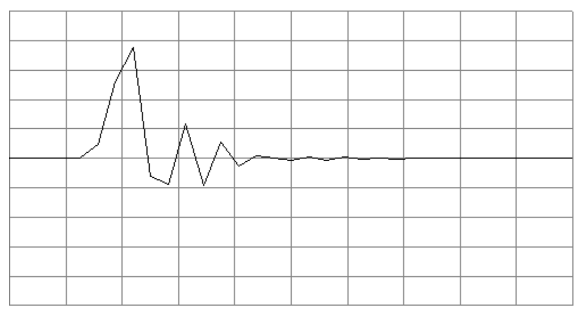

Figure-2, Impulse response from finite differences, 0.5V/div vertical, 1.25us/div horizontal.

The frequency response can be computed from the step response by computing its Fourier transform. Figure-3 shows the magnitude of the FFT of the measured impulse responses zero padded to a block length of 1024. Only the first 32 samples of the impulse response were included in this block. The reason for zero padding to a long block length is to provide more points in the frequency response for easier visualization. Several individual measurements were made with each frequency response plotted on top of one another in the graph. The thickness of the resulting line shows the variability of the computed frequency response caused by changing step arrival time relative to the ADC sample times. Note that the variability is quite large near the upper frequency end of the graph. That is because the upper portion of the spectrum is not fully alias protected at the selected sample rate and bandwidth combination (1MHz bandwidth, 2.56MHz sample rate). The transition band of the digital filters extends from 1MHz to 1.56MHz. Thus the frequency band of 1.28MHz-1.56MHz folds back onto the 1MHz-1.28MHz band. With different step arrival times these two "aliases" will either constructively or destructively interfere, causing the variability seen in the graph. Because of this uncertainty, the portion of the graph from 1MHz to 1.28MHz is not used and will not be included in subsequent computations.

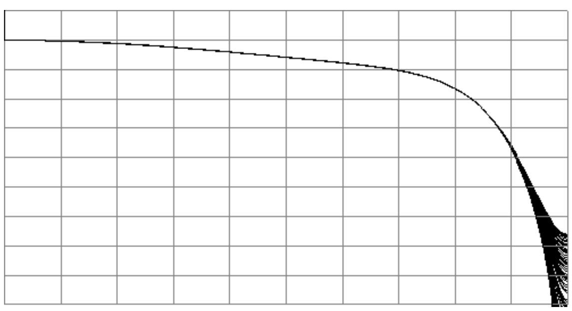

Figure-3, Frequency response magnitude, 0-1.28MHz, 2dB/div vertical, multiple unaveraged measurements.

Another noticeable feature of figure-3 is the general roll-off of the frequency response. This is a result of using the finite difference approximation of the derivative to compute the impulse response from the step response. Figure-4 shows the same frequency response after being adjusted for the sin(x)/x response of this finite difference approximation. Note that the vertical scale has been expanded by a factor of twenty to .1dB per division. The resulting response looks very similar to the digital filter response graphed in the E1437A data sheet.

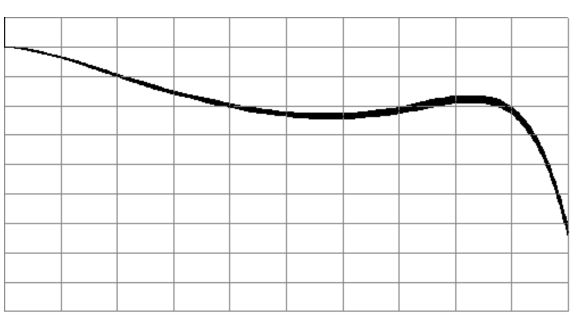

Figure-4, Frequency response magnitude, sin(x)/x removed, 0-1MHz, 0.1dB/div vertical, multiple unaveraged measurements.

Since the digital filter response is perfectly repeatable, it can be compensated for in the measurement. To make this easier, the E1437A library includes the C-function called hpe1437_filter_resp_get( ). Figure-5 shows a family of individually measured frequency responses after removing the effects of the digital filter and the nominal analog filter. The vertical scale has once again been expanded to 0.02dB per division.

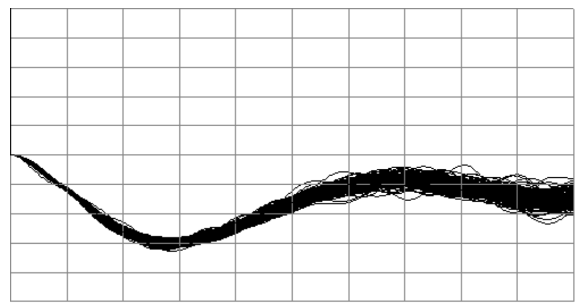

Figure-6, Frequency response magnitude, sin(x)/x and nominal E1437A filter response removed, 0-1MHz, 0.02dB/div vertical, unaveraged multiple measurements.

With the expanded vertical scale we can now see the effect of measurement noise. The thickness of the line in figure-6 is a result of this noise, not of the aliasing problem discussed earlier. If more accuracy is required, several individual response measurements can be averaged together as shown in figure-7. Even with averaging 100 individual measurements, the total measurement time can be quite short. With a sufficiently fast signal processor to do the computations these 100 averages could be done in 100ms since the steps are repeated at a 1kHz rate. This is a very fast way to achieve better than .005dB repeatable frequency response results.

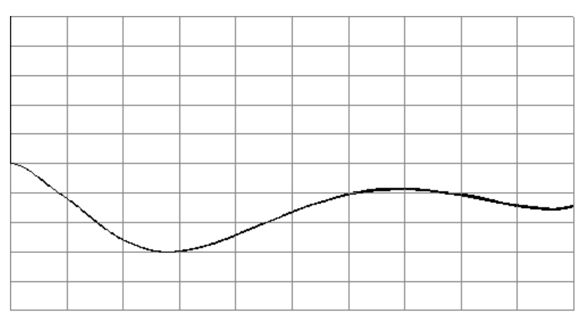

Figure-6, Frequency response magnitude, sin(x)/x and nominal E1437A filter response removed, 0-1MHz, 0.02dB/div vertical, multiple measurements with 100 averages per measurement.

The repeatable response plot shown in figure-6 is dominated by the "non-flatness" of the step source. This can be demonstrated by using a different step source. Figure-7 shows the same measurement using an HP 3326A Two Channel Function Generator instead of the 8131A Pulse Generator. The different response graphs indicate that the voltage steps produced by these two sources deviate from ideal in a different way. However, each source produces a very repeatable "almost ideal" step function.

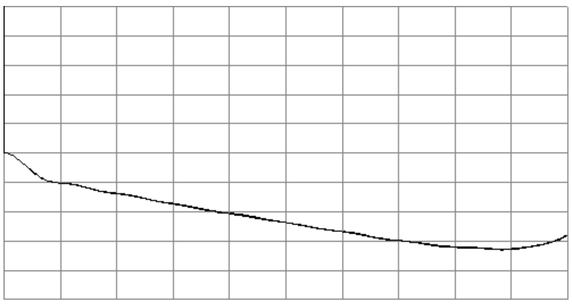

Figure-7, Same setup as figure-6, using 3326A as step source.

If the ultimate accuracy frequency response measurement is required, the source non-flatness could be measured and stored. It could then be used to compensate subsequent response measurements. The resulting inaccuracy would then be primarily due to the variance caused by measurement noise. In the example shown this is approximately ±0.01dB for unaveraged single-shot measurements. Averaging improves this by a factor equal to the square root of the number of averages used.

When using the E1437A for direct step response measurement of frequency response, the following key points should be remembered.

- Direct step response measurement of frequency response can be done with inexpensive and readily available signal sources which do not have to be integrated with the measurement "instrument".

- Both magnitude and phase information can be obtained without having to use a reference path. This is important for end-to-end measurements.

- Very good results can be achieved with a single step response. This is probably the fastest way possible to characterize a linear system response.

- The sampled measurement must be alias free over the frequency range of interest. The E1437A provides for this with its built-in filters.

- Low noise and good linearity are necessary for accurate results. The E1437A provides both.